# Did St Augustine anticipate “Moore’s Principle”?

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**Abstract**
This paper deals with the question whether in his *De mendacio* the church father Augustine anticipated what in modern times came to be referred to as “Moore’s Principle”. This law of epistemic logic says that a person is *convinced* that *p*, (if and) only if she *believes* that she *knows* that *p*. The present investigation makes use of the formal apparatus of modern epistemic logic as sketched in section 2. The main results of epistemic logic in the later Middle Ages will be summarized in section 3. In particular, it will turn out that a variant of “Moore’s principle” was endorsed by the 14th century logician William Heytesbury in his treatise *De Scire et Dubitare*. Augustine’s theory of lying and his views concerning the various forms of believing, knowing, and doubting are scrutinized in section 4. If one assumes that the manuscript of *De mendacio* contains two scribal errors, Augustine recognized that a person believes to know (“putat se scire”) that *p* just in case that she firmly believes (“firmissime credit”) that *p* and hence has no doubts at all (“omnino non dubitet”) that *p*.

**Keywords:** Epistemic logic; Augustine; Heytesbury; “Moore’s principle”.

## 1. Introduction

At the 24th European Symposium on Medieval Logic and Semantics (Parma, 2024), Roberto Limonta presented a paper on Augustine’s theory of lying. As I learnt from his presentation, the church father Augustine (354–430) pointed out in his *De mendacio liber unus* that not everybody saying something *false* must therefore be a liar. In particular, one does not lie if one says something which one *believes* to be true (“Non enim omnis qui falsum dicit mentitur, si credit aut opinatur verum esse quod dicit.”) Immediately after this explanation, Augustine makes an interesting and profound remark concerning the logical relationships between the epistemic concepts of belief, knowledge, ignorance, and doubt:

> Inter credere autem atque opinari hoc distat, quod aliquando ille qui credit, sentit se ignorare quod credit, quamvis de re quam se ignorare novit omnino non dubitet, si eam firmissime credit; qui autem opinatur, putat se scire quod nescit. (Augustine (1990), section 3.3)

An English translation roughly runs as follows:

> Between believing and having an opinion there is this difference, that sometimes he who believes feels that he does not know what he believes, although he has no doubts at all about what he knows that he doesn't know, if he believes it in the strongest form. But he, who has an opinion, believes that he knows that which he doesn't know.

The main aim of this paper is to find out whether Augustine here anticipated what in modern times came to be referred to as "Moore's Principle", i.e., the law of epistemic logic according to which a person *a* is *convinced* that *p*, (if and) only if she *believes* that she *knows* that *p*. For this purpose, the formal apparatus of *modern epistemic logic* will be sketched in section 2. The main results concerning *Epistemic logic in the later Middle Ages* will be summarized in section 3. Finally, Augustine's theory of lying and his views concerning the various forms of believing, knowing, and doubting will be scrutinized in section 4. In an appendix, the history of the "re-discovery" of "Moore's principle" in the 20th century will be outlined.

## 2. Modern epistemic logic

In the wake of Jaakko Hintikka's pioneering works, the epistemic/doxastic attitudes '*a* knows that *p*' and '*a* believes that *p*' are usually symbolized by '$K_a p$' and '$B_a p$', respectively. Here, we use the variants '$K(a,p)$' and '$B(a,p)$' which better indicate that the attitudes are *relations* between a subject *a* and a proposition *p*. Hintikka characterized his *Knowledge and Belief* as "An Introduction to the logic of the *two* notions". As has been argued in Lenzen (1975), however, it seems better to construct epistemic/doxastic logic as the theory of (at least) *three* notions because one has to distinguish between "strong" and "weak" belief.

While knowledge is apparently a matter of yes or no, belief is a matter of *degrees*. Beliefs can range from rather uncertain presumptions up to firm convictions.[1] With the help of a function $\mathrm{Prob}_a(p)$ which assigns, for each proposition $p$, a real number $r$ from the interval $0 \leq r \leq 1$ denoting the *probability* which $p$ has for subject $a$, one can define that $a$ is (absolutely) *convinced* that $p$ by:

DEF 1 $C(a,p) =_{\mathrm{df}} \mathrm{Prob}_a(p) = 1$.

Thus, conviction is sort of *doxastic necessity*. Now just as, in the field or alethic modal logic, a proposition $p$ is *possible* iff it is not necessarily false, $\Diamond p \Leftrightarrow_{\mathrm{df}} \neg\Box\neg p$, one can introduce a corresponding notion of *doxastic possibility* by:

DEF 2 $P(a,p) =_{\mathrm{df}} \neg C(a,\neg p)$.

In probabilistic terms, $P(a,p)$ means that the probability which person $a$ assigns to proposition $p$ (or, perhaps better, to the state of affairs described by $p$) is *greater than 0*. Therefore, one may appropriately interpret '$P(a,p)$' as saying that *a considers p as possible*.[2]

Of course, $a$ may *believe* that $p$ (in a sense weaker than $C(a,p)$) even if $\mathrm{Prob}_a(p) < 1$. As a *minimal* condition for such a "weak" belief, one apparently has to require that *$p$ is more probable than $\neg p$*, or in quantitative terms, that the probability of $p$ for $a$ must be *greater than ½*. For the sake of simplicity, let us assume that this *necessary* condition is also *sufficient* for $a$'s ("weakly") believing that $p$, so that one can define:

DEF 3 $B(a,p) =_{\mathrm{df}} \mathrm{Prob}_a(p) > ½$.

According to the standard theory of probability, the probability of the *negation* of $p$, $\mathrm{Prob}_a(\neg p)$, equals $(1 - \mathrm{Prob}_a(p))$.[3] Therefore, if $p$ is believed by $a$, $\neg p$ cannot be believed by $a$ as well. More generally, for probabilistic reasons, the following chain of doxastic entailments holds:

[1] Thus, O'Connor (1968) p. 1 remarked: "I can believe a given proposition with varying degrees of assent".

[2] Our '$P(a,p)$' must not be mixed up with Hintikka's '$P_a p$' which is equivalent to $\neg K_a \neg p$. Cf. also fn. 9 below.

[3] The theory of subjective probability was founded in particular by B. de Finetti; for details cf. De Finetti (1964) and Kutschera (1972).

D1 $C(a,p) \Rightarrow B(a,p) \Rightarrow \neg B(a,\neg p) \Rightarrow P(a,p)$.[4]

Furthermore, the probability of a *conjunction*, $\text{Prob}_a(p \wedge q)$, is at most as great as the *product* of the single probabilities, $\text{Prob}_a(p) * \text{Prob}_a(q)$. Therefore, it may happen that even though person $a$ believes both $p$ and $q$, $B(a,p) \wedge B(a,q)$, $a$ does not believe that $(p \wedge q)$ because the probability of $(p \wedge q)$ is < ½. The concept of "strong" belief, however, does satisfy the law of conjunction $C(a,p) \wedge C(a,q) \Rightarrow C(a,p \wedge q))$.[5]

*Knowledge* differs from belief primarily in that only what is *true* can be known (by $a$):

K1 $K(a,p) \rightarrow p$.

In contrast, a belief – even in the form of a firm conviction – may always turn out to be mistaken, i.e., the subsequent principles are *invalid* (as signalled by the added '*')

D2* $C(a,p) \rightarrow p$
D3* $B(a,p) \rightarrow p$.

Ever since Plato's classical analysis of knowledge as true, justified belief, $a$'s knowing that $p$ is taken to presuppose $a$'s believing that $p$. In the modern literature this "*Entailment thesis*" is usually formalized as:

K2 $K(a,p) \rightarrow B(a,p)$.

However, in the light of our distinction between "strong" and "weak" belief, the following variant better expresses the idea that $a$ cannot know that $p$ unless $a$ is also convinced that $p$:

K3 $K(a,p) \rightarrow C(a,p)$.[6]

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4 Clearly, $\text{Prob}_a(p) = 1$ entails $\text{Prob}_a(p) > ½$, which in turn entails $\text{Prob}_a(p) \geq ½$, which finally entails $\text{Prob}_a(p) > 0$. Thus, in particular, the *laws of consistency* $C(a,p) \rightarrow \neg C(a,\neg p)$ and $B(a,p) \rightarrow \neg B(a,\neg p)$ are probabilistically valid.

5 As has been shown in Lenzen (1980), the conjunction principle holds only for the strongest form of belief, $C(a,p)$, but neither for "weak" belief, $B(a,p)$, nor for any other "medium" belief as it might be defined by $B_r(a,p) =_{\text{df}} \text{Prob}_a(p) > r$ (where $1 < r \leq ½$ ).

6 This doesn't mean that K2 is wrong; just to the contrary, K2 follows from K3 on account of the doxastic relations summarized in D1.

According to a popular philosophical thesis, human subjects have a “privileged access” to their own mental states, which means, roughly, that although one may be mistaken with respect to someone else’s beliefs, desires, fears, etc., such errors are excluded when one’s own mental attitudes are at stake. This principle is apt to justify the subsequent *iteration principles* according to which *a* always *knows* whether she believes (or disbelieves) a proposition *p*:

D4 $C(a,p) \rightarrow K(a,C(a,p))$
D5 $B(a,p) \rightarrow K(a,B(a,p))$
D6 $\neg C(a,p) \rightarrow K(a,\neg C(a,p))$
D7 $\neg B(a,p) \rightarrow K(a,\neg B(a,p))$.[7]

In view of K3 and D1, one immediately obtains the following corollaries saying that if person *a* (“strongly” or “weakly”) *believes* (or disbelieves) *that p*, she also (“strongly” or “weakly”) *believes that she believes* (or disbelieves) that *p*:

D8 $C(a,p) \rightarrow C(a,C(a,p))$
D9 $\neg C(a,p) \rightarrow C(a,\neg C(a,p))$
D10 $B(a,p) \rightarrow B(a,B(a,p))$
D11 $\neg B(a,p) \rightarrow B(a,\neg B(a,p))$.

From this it further follows that *a* (“strongly” or “weakly”) *believes* that *p if and only if* she (“strongly” or “weakly”) *believes that she believes* that *p*:

D12 $C(a,p) \leftrightarrow C(a,C(a,p))$
D13 $B(a,p) \leftrightarrow B(a,B(a,p))$.[8]

Let’s now turn to knowledge! The so-called “KK-Thesis” says that whoever knows that *p*, thereby already knows that she or he knows that *p*:

K4 $K(a,p) \rightarrow K(a,K(a,p))$.

[7] In view of K1, all these implications could be strengthened into bi-conditionals. For some rather strange reasons, Hintikka doubted the validity not only of D5, but even of the corollary $B(a,p) \rightarrow \neg K(a,\neg B(a,p))$, arguing that “there does not seem to be any reason why one can believe only things which are known to be possible (according to what one knows)” (Hintikka (1962), p. 53).

[8] To show that, e.g., $B(a,B(a,p))$ entails $B(a,p)$, assume that the latter is not the case, $\neg B(a,p)$. According to D11, one then gets $B(a,\neg B(a,p))$, and because of D1 further $\neg B(a,\neg\neg B(a,p))$, i.e., $\neg B(a,B(a,p))$, in contradiction to premise $B(a,B(a,p))$.

In view of truth-axiom K1, this principle can be strengthened into a bi-conditional:

K5 $K(a,p) \leftrightarrow K(a,K(a,p))$.

Accordingly, the chain of doxastic entailment, D1, can be extended to the subsequent chain of epistemic/doxastic entailments:

E1 $K(a,p) \Rightarrow C(a,p) \Rightarrow B(a,p) \Rightarrow \neg B(a,\neg p) \Rightarrow P(a,p) \Rightarrow \neg K(a,\neg p)$.[9]

Now, unlike in the case of *belief*, the concept of *knowledge* does not fully satisfy the principle of "negative introspection"[10]:

K6* $\neg K(a,p) \rightarrow K(a,\neg K(a,p))$.

In general, the falsity of '$K(a,p)$' can result either from *subjective* or from *objective* reasons. In the former case, i.e. when person *a* does *not* (strongly) *believe* that *p*, then – as maintained in D7, D6 – she *knows* that she *doesn't believe* that *p*, so that she *knows* a fortiori that she *doesn't know* that *p*:

E2 $\neg C(a,p) \rightarrow K(a,\neg K(a,p))$
E3 $\neg B(a,p) \rightarrow K(a,\neg K(a,p))$.

However, if the failure to know that *p* is due to the objective reason that *p* happens to be *false* (while *a* strongly *believes p* to be true), then it would be entirely unreasonable to postulate that *a knows* that she *doesn't know* that *p*. For, according to "Moore's principle", whenever person *a* is *certain* that *p*, she *believes that she knows* that *p*:

E4 $C(a,p) \rightarrow B(a,K(a,p))$.

[9] Apparently, there is no "natural" expression for this *epistemic* attitude in English (or in any other natural language known to me). Hintikka symbolized it as '$P_a(p)$' and paraphrased it as 'it is possible, for all that *a* knows, that *p*'. Note also that our *doxastic* possibility operator $P(a,p)$ was symbolized by Hintikka as '$C_ap$'.
[10] The expression 'negative introspection' is used in particular in Lamarre & Shoham (1994), p. 415: "2. Negative introspection– e.g., 'if John does not believe then he believes that he does not believe' – is an acceptable idealization for belief and certainty, but not for knowledge."

But, according to E1, $B(a,K(a,p))$ entails $\neg K(a,\neg K(a,p))$, so that we have a clear instance of $\neg K(a,p)$ (since $p$ was assumed to be false), and yet $\neg K(a,\neg K(a,p))$.

Let it be mentioned in passing that E4 may be strengthened into the equivalence

E5 $\quad C(a,p) \leftrightarrow B(a,K(a,p))$,

and that besides this "Moorean" principle, also the following "Lenzen's law" becomes provable:

E6 $\quad C(a,p) \leftrightarrow \neg K(a,\neg K(a,p))$.[11]

To conclude this section, let us briefly consider the question whether, or to which degree, epistemic and doxastic attitudes are *closed under logical entailments*. If the task of epistemic logic is viewed as modelling the structural relations of an *idealized rational subject*, then the subsequent principles of so-called "logical omniscience" and "logical omni-belief" appear quite acceptable:

K7 $\quad$ If $(p \Rightarrow q)$, then $(K(a,p) \rightarrow K(a,q))$
D14 $\quad$ If $(p \Rightarrow q)$, then $(C(a,p) \rightarrow C(a,q))$
D15 $\quad$ If $(p \Rightarrow q)$, then $(B(a,p) \rightarrow B(a,q))$.[12]

However, if the task of epistemic logic is considered as describing the structure of concrete human beings, then these rules appear much too strong. Thus, with respect to K7, Hintikka emphasized in (1962), pp. 30–31, that "it is clearly inadmissible to infer 'he knows that *q*' from 'he knows that *p*' solely on the basis that *q* follows logically from *p*, for the person in question may fail to see that *p* entails *q*, particularly if *p* and *q* are relatively complicated statements". Therefore, it seems reasonable to require that the corresponding implications only hold if the subject *a knows* that $(p \Rightarrow q)$:

K8 $\quad$ If $K(a,(p \Rightarrow q))$, then $(K(a,p) \rightarrow K(a,q))$
D16 $\quad$ If $K(a,(p \Rightarrow q))$, then $(C(a,p) \rightarrow C(a,q))$
D17 $\quad$ If $K(a,(p \Rightarrow q))$, then $(B(a,p) \rightarrow B(a,q))$.

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[11] A closer discussion of these principles is to be found in the Appendix.
[12] Such strong rationality assumptions are accepted in particular by people working in the field of "knowledge representation" in artificial systems.

As will be shown in the next section, most medieval logicians favoured restricted principles.

## 3. Medieval epistemic logic

In the standard work Boh (1993), the main achievements of *Epistemic logic in the Later Middle Ages* were summarized in twelve items, of which for our purposes the following five are relevant:[13]

> (iii) a search for firm, demonstrative knowledge of *necessary propositions* by Grosseteste, […} leading to a distinction of various senses of *scire* and eventually leading to a search of the necessary and sufficient conditions for knowing *contingent propositions* by the authors of the *de scire et dubitare* literature, such as Kilvington, and Heytesbury;
> (iv) a search – especially among the early theologians such as Anselm, Albert the Great, Thomas Aquinas, etc. – for the proper conceptual relationship of knowing, believing, having conviction, having faith, and truth; […]
> (ix) an attempt in the later fourteenth century (e.g. Strode, Peter of Mantua) to systematize the most general principles of epistemic logic and to co-ordinate them with alethic and obligational principles, and of course with the principles of propositional logic; […]
> (xi) a recognition that there is a sort of analogy between assertable principles in one realm, e.g. $\Box p \rightarrow p$ and $K(a,p) \rightarrow p$, and the rejected claims, such as $* \; p \rightarrow \Box p$ and $* \; p \rightarrow K(a,p)$ […];
> (xii) recognition of iterated epistemic/doxastic modalities – in a philosophical context – by thinkers such as Albert the Great and Thomas Aquinas, and – in reflective logical contexts – by logicians such as Heytesbury, Gaetanus of Thiene, and Frachantian […]. (Boh (1993), 127–129).

Let it be noted that Boh uses our '$K(a,p)$' and '$B(a,p)$' to symbolize '$a$ knows that $p$' and '$a$ ("weakly") believes that $p$', respectively. Furthermore, he symbolizes $a$'s "strong" belief by '$B^*(a,p)$', but in what follows this formula

[13] Another interesting complex mentioned by Boh concerns the "(vi) discovery of special, Gettier-like problems and epistemic paradoxes". Unfortunately, this topic lies beyond the scope of this paper.

shall be replaced by our ‘$C(a,p)$’.[14] We do, however, adopt Boh’s additional ‘$D(a,p)$’ for ‘$a$ doubts that $p$’, but it remains to be discussed how this operator is to be defined exactly.

### 3.1. Knowledge, belief, necessity, and truth

Many medieval logicians regarded epistemic/doxastic modalities like being *known*, being *believed*, or being *doubted* as basically on a par with the traditional “modes” of being *necessary*, *possible*, *impossible*, and *contingent*. The logical relations among the *alethic* modalities were well-known in the Middle Ages. Thus, in his *Summulae Logicales*, Peter of Spain (1205–1277) draw the following diagram:

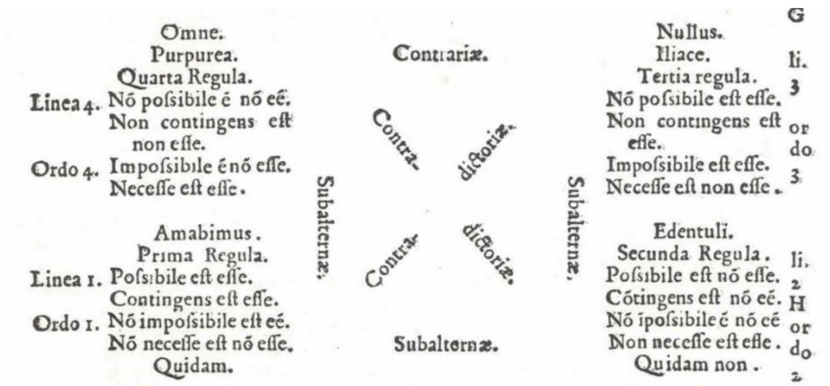


*Fig. 1 Peter of Spain’s Square of Modal Opposition*[15]

This figure displays the parallel between the usual square of opposition for *categorical* propositions (briefly alluded to by the words ‘Omne’, ‘Nullus’, ‘Quidam’ and ‘Quidam non’) and the square of opposition for *modal* propositions which rests on an analogy between the quantifiers ‘every’, ‘no’, and ‘some’ on the one hand, and the modalities ‘necessary’, ‘impossible’, and ‘possible’ on the other hand.[16] Furthermore, Peter paraphrased the modal

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[14] Boh also uses a symbol ‘$C(a,p)$’, but with a very different meaning, namely to abbreviate that subject *a considers* proposition *p*. This propositional attitude, as well as, e.g., ‘*a understands* that *p*’, can stay out of consideration here.
[15] The scan has been taken from p. 42B of Petrus Hispanus (1572).
[16] This analogy can be seen as an anticipation of possible-worlds-semantics according to which a necessary proposition is true in every world; a possible proposition true in at least one world, and an impossible proposition true in no world.

propositions in each of the four corners in (at least) three different ways. In particular, the proposition in the upper left corner is not only formulated as '*p* is *necessary'*, but also as 'not-*p* is *impossible'* and as 'not-*p* isn't *possible'*. This shows that Peter was well aware of the standard laws:

A1 $\Box p \leftrightarrow \neg\Diamond\neg p$
A2 $\Diamond p \leftrightarrow \neg\Box\neg p$.[17]

In addition to A1 and A2, medieval logicians endorsed the principles that every necessary proposition is true, and every true proposition possible:

A3 $\Box p \rightarrow p$
A4 $p \rightarrow \Diamond p$.

A3 further entails that no false proposition can be necessary:

A5 $\neg p \rightarrow \neg\Box p$.

When interpreted *epistemically*, A3 says, as captured by principle K1, that if *a* knows that *p*, *p* must in fact be *true*.[18] Accordingly, A5 says that if *p* is false, then *a* can't know that *p*:

K9 $\neg p \rightarrow \neg K(a,p)$.

The medieval slogan "Nothing but the true is known" (Burley (2000), p. 156) summarizes the laws K1 plus K9 in an elegant way. As a corollary of K1 and A4, one further obtains:

K10 $K(a,p) \rightarrow \Diamond p$.

[17] Interestingly, these laws have a "natural" counterpart only in the field of *doxastic* attitudes, where $C(a,p) \leftrightarrow \neg P(a,\neg p)$ says that *a* is *convinced* that *p* iff *a doesn't consider* it as *possible* that not-*p*; similarly, DEF 1, i.e., $P(a,p) \leftrightarrow \neg C(a,\neg p)$, holds that *a* considers it as possible that *p* iff *a* is not convinced that not-*p*. In the field of *epistemic* logic, however, no corresponding "natural" laws exist. It is true, though, that Hintikka introduced a somewhat artificial operator $P_a p$ to abbreviate that "*p* is possible according to everything that *a* knows". $P_a p$ can hence be defined as $\neg K_a\neg p$, so that $K_a p \leftrightarrow \neg P_a\neg p$; but these equivalences apparently don't express "natural" relations of ordinary language.
[18] Principle K1 had been put forward already by Petrus Hispanus. Cf. Peter of Spain (1572), p. 127: "Quidquid scitur est verum".

This principle was put forward by Ockham by stating that “every known proposition is a possible proposition”.[19] Interestingly, a corresponding principle is maintained by Ockham to hold also in the case of *belief*. No matter whether the belief is right or wrong, the believed proposition must at least be *possible* (“omnis propositio credita est propositio possibilis”):

D18 $B(a,p) \rightarrow \Diamond p$
D19 $C(a,p) \rightarrow \Diamond p$.

As expressed by our earlier principles D2* and D3*, however, the *doxastic* counterparts of the “truth-axiom” K1, are clearly invalid, no matter whether ‘belief’ is interpreted in the “weak” or “strong” sense. This has been pointed out, rather incidentally, by Ockham who briefly discussed the question which modalities entail, and which are entailed by, corresponding de-modalized propositions (“de inesse”). On the one hand, the truth of ‘A man is white’ doesn’t suffice to conclude ‘A man is known to be white’. More generally, the *converse* of K1 is invalid:

K11* $p \rightarrow K(a,p)$.[20]

On the other hand, if a proposition as, e.g., ‘A man is white’ is *believed* by Socrates to be true, it doesn’t follow that this proposition must therefore be true.[21] More generally, Ockham stated the following rule:

> If a modality is such that it only applies to true propositions [as ‘is necessary’, ‘is known’], the inference from the modalized proposition to the simple proposition *de inesse* is valid. But if the modality is such that it could be applied to a false proposition, the inference from the

[19] Cf. Ockham (1974), p. 641: “[...] omnis propositio scita est propositio possibilis”. Ockham almost always formulates propositional attitudes in an impersonal form such as ‘scitur’, ‘creditur’, ‘opinatur’, etc. Hence it is not entirely correct to formalize ‘propositio scita’ as the personalized variant ‘$K(a,p)$’. For a rare example of a personal belief of type $B(a,p)$ cf. fn. 21 below.

[20] Cf. Ockham (1974), pp. 638–639: “Circa alias modales sciendum quod raro illae de inesse inferunt illas de modo; sicut non sequitur […] ‘homo est albus, ergo homo scitur esse albus’ [...] Tamen frequenter illae de modo inferunt illas de inesse; sicut sequitur [...] ‘album scitur esse homo, igitur album est homo’”. Principle K11* expresses the idea that subject *a* is *omniscient*.

[21] Cf. Ockham (1974), p. 639: “Aliquae tamen non inferunt suas de inesse; sicut non sequitur ‘Sortes creditur esse albus, igitur Sortes est albus’; nec sequitur ‘Sortes opinatur hominem esse album, igitur homo est albus’”. The main difference between these two examples is that in the former case Socrates is the *object* of an *de re* belief, namely, he is beliveved (by someone) to be white, while in the latter case Socrates is the *subject* of a de dicto belief, namely, he believes that some man is white.

> modalized proposition to the simple proposition *de inesse* is not valid. Such modalities are 'is believed', 'is opiniated', […] and so on.[22]

Ockham mentions yet another law relating alethic and doxastic attitudes by noting that 'necessarily' and 'unbelievable' are "repugnant" to each other.[23] Although, apparently, '$p$ is necessary' doesn't suffice to conclude that $p$ is *believed* (by someone, $a$); one can at least conclude that $p$ is *believable*:

D20 $\Box p \rightarrow \Diamond C(a,p)$
D21 $\Box p \rightarrow \Diamond B(a,p)$.

Similarly, the necessity of $p$ may perhaps not suffice to conclude that $p$ is *known*, but it warrants at least that $p$ is *knowable*:

K12 $\Box p \rightarrow \Diamond K(a,p)$.[24]

**3.2. Does Knowledge entail Belief?**

According to Boh, the English theologian and philosopher Robert Grosseteste (ca. 1175–1253) distinguished four types of knowledge:

(i) to know in a broad sense (*scire communiter*) – 'a comprehension of truth, and in this sense fallible contingents are known';
(ii) to know properly (*scire proprie*) – 'comprehension of truth of those things which happen always or frequently in the same way, and in this sense natural events are known, […]';
(iii) to know more properly (*scire magis proprie*) – 'a comprehension of truth of those things which are always disposed in one and the same way; and in this sense [in mathematics] the principles and the conclusions are known'; […]

[22] Cf. Ockham (1974), p. 639: "[…] si sit talis modus qui non potest competere nisi propositioni verae, consequentia est bona ab illa de modo ad suam de inesse. Si autem sit talis modus qui possit competere propositioni falsae, consequentia ab illa de modo ad suam de inesse non valet. Huiusmodi sunt tales: creditum, opiniatum, concessum, dubitatum et huiusmodi".

[23] Cf. Ockham (1974), p. 642: "Oportet autem scire quod modorum quidam sunt repugnantes, quidam sunt secundum superius et inferius se habentes, quidam sunt disparati. Repugnantes sunt necessarium et impossibile, [...] necessarium et inopinabile [...] et tales multi". In general, two propositions are "repugnant" to each other iff the truth of one proposition entails the falsity of the other.

[24] Cf. Ockham (1974), pp. 642–3: "Secundum superius et inferius se habentes sunt necessarium et possibile; nam omne necessarium est possibile, et non e converso. [...] Similiter se habent necessarium et scibile".

(iv) to know most properly (*scire maxime proprie*) – 'as comprehension of that which is [known] immutably by comprehending that from which it has immutable being […]'.[25]

While the IVth type of knowledge is apparently concerned with *theological* issues, type III deals with propositions which are mathematically true and hence *formally necessary*. Similarly, knowledge of type II is concerned with laws of nature, i.e., with *nomologically necessary* propositions. Only type-I knowledge has to do with non-necessary or *contingent* propositions. Here one may suppose that besides $K(a,p)$, also $K(a,\Diamond\neg p)$ holds.

Grosseteste further distinguished between "common belief", "proper belief", and "more proper belief".[26] The first type (*opinio communiter*) is defined as "'a cognition with assent, and in this sense it is the same as faith (*fides*)'". One may plausibly assume that this notion coincides with our broadly conceived '$B(a,p)$' which includes the special case '$C(a,p)$'. Grosseteste (1981), p. 19 explicitly maintained that everything which is *known* is also *believed* in this sense ("quicquid scitur opinatur hoc modo"). Hence, he would probably assent both to K2 and to K3.

The second type (*opinio proprie*) is described as "'an acceptance of one part of a contradiction with a fear of (the possibility of) the opposite'". This means that, given two opposite propositions $p$ and $\neg p$, subject *a believes* that $p$ but *considers it as possible* that not-$p$. Boh tried to formalize this by the formula '$B(a,p) \wedge B(a,\Diamond\neg p)$'.[27] It seems more appropriate, however, to formalize it as '$B(a,p) \wedge P(a,\neg p)$', i.e., as an instance of a "weak" belief which fails to satisfy the condition of "strong" belief. According to Grosseteste, knowledge does *not* entail belief in *this* sense ("secundum hoc non est scientia opinio"). One may plausibly assume that this claim is based on the view, expressed by our K3, that *knowledge presupposes conviction*. Clearly, the *validity* of '$K(a,p) \rightarrow C(a,p)$' entails the *invalidity* of '$K(a,p) \rightarrow (B(a,p) \wedge \neg C(a,p))$'. Grosseteste apparently had such considerations in mind, for he explained:

> Thus, it is evident that it cannot happen that one and the same person simultaneously believes and knows the same, because, if it is known, it

[25] Cf. Boh (1993), pp. 23–24; our addition of 'in mathematics' is required by the original text which runs: "et sic sciuntur in mathematicis tam principia quam conclusiones"; the quotations from Grosseteste (1981) are to be found in Boh's endnotes, p. 139.

[26] Cf. Grosseteste (1981), p. 19: "Opinio tripliciter dicitur, communiter, scilicet, proprie, et magis proprie".

[27] Cf. Boh (1993), p. 27. Boh's symbols '&' and '~' have been replaced by '$\wedge$' and '$\neg$'.

> is judged that it could not be otherwise; [but] if it is believed, it is judged that it can also be otherwise.[28]

The third kind (*opinio magis proprie*) is characterized as "an acceptance of an immediate proposition which is not necessary, and this is opinion-more-properly-so-called". Grosseteste doesn't explicitly say whether such a belief is entailed by knowledge, or not. He only emphasis that the object of such beliefs are contingent and unprovable "things", in contrast to what the sciences deal with.[29]

Summing up, when Grosseteste discusses the logical relations between knowledge and belief, he makes a major distinction between (scientific) knowledge of *necessary* truths and common knowledge of *contingent* facts. The latter may be formally characterized as '$K(a,p) \wedge K(a,\Diamond\neg p)$'. Furthermore, he distinguishes between common belief $B(a,p)$ (which includes the special case $C(a,p)$), and "proper" belief which obtains when $B(a,p)$ but not $C(a,p)$. Grosseteste apparently endorsed both K2 and K3, but since knowledge requires certainty, knowledge is incompatible with a "proper", i.e., a "weak" but *not* "strong" belief.

### 3.3. Logical closure of knowledge

Most medieval logicians believed that a logical entailment between $p$ and $q$ doesn't suffice to conclude that, if $p$ is known (by $a$), $q$ must be known as well. Thus, William Ockham (1287–1347) maintained that even the following rule, where $p$ and $q$ are supposed to *mutually entail each other*, is not generally valid:

K13* If $(p \Leftrightarrow q)$, then $(K(a,p) \rightarrow K(a,q))$.

According to Ockham, "it is possible that one of a pair of convertibles be known, even though the other one is not known, indeed the other *may not even come to mind*".[30] Ralph Strode (ca. 1350–1400) might appear to have been less sceptical, for in his theory of consequences one finds the rule: "If a

[28] Cf. Grosseteste (1981), p. 19: "Ex his manifestum est quod non contingit eundem hominem simul opinari et scire idem, quia, si scita, arbitratur hoc non posse aliter se habere; si opinatur, arbitratur hoc posse et aliter esse".
[29] Cf. Grosseteste (1981), p. 19: "Opinio […] est acceptio inmediate propositionis non necessariae, supple, est indemonstrabilis, et in hoc differt a scientia".
[30] Cf. Boh (1993), p. 54; my emphasis.

consequence is formally valid, and if the antecedent is known, therefore also the consequent is known".[31]

K14* If $(p \Rightarrow q)$, then $(K(a,p) \rightarrow K(a,q))$.

But, according to Boh (1993), p. 150, Strode explained later on that one must *know* that the consequence is sound. At any rate, Paul of Pergula (ca. 1400–1455) explicitly postulated the weaker rule: "If a consequence is valid and known by you to be valid, and if the antecedent is known by you, also the consequent is known by you":

K15 If $K(a,p \Rightarrow q)$, then $(K(a,p) \rightarrow K(a,q))$.[32]

On the other hand, some medieval logicians occasionally envisaged stronger principles for the logical closure of someone's knowledge and/or belief. Thus, in chapter 30 of part III–1 of the *Summa Logicae*, Ockham appears to endorse principle D14 or D15, when he mentions the "true rule": 'the premisses are believable, therefore the conclusion is believable'. Furthermore, Ockham rejects a corresponding principle for *knowledge* ("premissae sunt scitae, igitur conclusio est scita") only because in the non-personalized version, formulated with 'scitum', it can happen that one premiss is known by some person *a* and the other premiss by another person *b*![33]

[31] Boh (1993), p. 150, note 35, mentions "Strode's thirteenth rule of consequence […]: 'Si aliqua consequentia est bona et formalis, et antecedens est scitum, ergo et consequens est scitum." Furthermore, on p. 158, note 18, Boh (1993) quotes from Strode's *Consequentie*: "'Si antecedens est scitum, consequens est scitum'".

[32] Some logicians were even more cautious and required that the consequence must not only be *known* to be sound, but that also the consequent *q* must be "understood" and "considered". Cf. Peter of Mantua's rule quoted in Boh (1993), p. 108: "'If there is a sound consequence … known [by a person] to be sound and its antecedent is known (*scitum*) and its consequent understood (mentally grasped, *intellectum*) and it is not incoherent (*non repugnat*) for the consequent to be known […] and he sufficiently considers (pays attention to, *considerat*) the consequent, then the consequent is known'".

[33] Cf. Ockham (1974), p. 436: "Et hoc quia talis discursus tenet per tales regulas veras 'praemissae sunt verae, igitur conclusio est vera'; 'praemissae sunt credibiles, igitur conclusio est credibilis' […]. Et hoc quia tales regulae falsae sunt 'praemissae sunt scitae, igitur conclusio est scita'; potest enim una praemissa sciri ab uno et alia ab alio et tamen ab utroque ignorari conclusio". A few lines later, Ockham characterizes the epistemic rule 'praemissae sunt scitae, igitur conclusio est scita' together with its doxastic counterpart 'praemissae sunt creditae, igitur conclusio est credita' as *false*.

### 3.4. Doxastic/epistemic interrelations

During the "seminal period of epistemic logic" in the 14th century, authors like Richard Kilvington (ca. 1300–1361) and William Heytesbury (ca. 1310–1372) composed treatises entitled *De Scire et Dubitare* which aimed to determine the necessary and sufficient conditions for knowledge. For this purpose, as Boh (1993), p. 63 put it, the "relation of concepts such as 'know', 'believe', 'firmly believe', 'unhesitatingly believe', 'opine', 'not know', 'doubt'" were explored. Nowadays, it is common to distinguish between knowing *that* and knowing *whether*, where the latter notion may be defined as:

DEF 3 $K_{\mathrm{wh}}(a,p) =_{\mathrm{df}} K(a,p) \vee K(a,\neg p)$.

Hence, '*a* does *not* know whether *p*' is equivalent to '*a* does *not* know that *p* *and a* does *not* know that not-*p*'. In a similar way, medieval logicians distinguished between corresponding senses of *doubting*, where, according to Boh (1993), p. 42, 'doubt' may either be "defined unilaterally as 'not know' or bilaterally as 'not know that *p* and not know that not-*p*'". With '$D(a,p)$' and '$D_{\mathrm{wh}}(a,p)$' abbreviating '*a* doubts *that p*' and '*a* doubts *whether p*', respectively, one can formalize these definitions as follows (where the '*' indicates that these principles are not entirely correct):

E7* $D(a,p) \leftrightarrow \neg K(a,p)$

E8* $D_{\mathrm{wh}}(a,p) \leftrightarrow \neg K(a,p) \wedge \neg K(a,\neg p)$.

The problem with E7*, E8* has been clearly recognized by Heytesbury who refuted an informal version of the right-to-left implication of E8* as follows:

> But it does not follow that if there is a proposition that someone […] does not know to be true or know to be false, that proposition is in doubt for him. For he may *firmly believe* […] in such a way that he unhesitatingly *believes that he knows* it – and it may nevertheless be false.

The idea behind this argument becomes even clearer when one considers the subsequent *example*:

> Suppose that someone who is not the king were to approach as the king ordinarily approaches, in similar circumstances, so that it would be generally said by everyone that he was the king; and suppose that he was like the king in all respects. I would so *firmly believe* that he was

> the king that I would *believe that I knew* that he was the king. And so, in that case I would not know that he was the king or that he was not the king, nor would I be in doubt whether he was the king.[34]

Hence, on the one hand, whenever someone *firmly* believes that *p*, he does *not* doubt that *p*:

E9 $C(a,p) \rightarrow \neg D(a,p)$,

or, by contraposition, if *a doubts* that *p*, *a* is *not certain* that *p*:

E10 $D(a,p) \rightarrow \neg C(a,p)$.

From this it further follows (by means of E2) that if *a doubts* that *p*, *a* doesn't *know* that *p*:

E11 $D(a,p) \rightarrow \neg K(a,p)$.

Hence, at least the left-to-right implication of E7* is correct. On the other hand, in the above example, *p* is *false*, so that *a can't know* that *p*. Hence, one has a clear instance of a case where $\neg K(a,p)$ (because of $\neg p$) and also $\neg K(a,\neg p)$ (because of $C(a,p)$) and yet $\neg D(a,p)$ and $\neg D_{wh}(a,p)$ (again because of $C(a,p)$).[35]

The most important point in Heytesbury's argument, however, is the clear endorsement of "Moore's principle" saying that whenever *a firmly* believes that *p*, *a believes* that he or she *knows* that *p*:

E12 $C(a,p) \rightarrow B(a,K(a,p))$.

In the following section we will have to see whether this principle had already been recognized, almost a millennium before Heytesbury, by the church father Augustine.

[34] Cf. Boh (1993), p. 71, my emphasis.

[35] The relations between doubting, believing, and knowing have also been investigated by other medieval logicians not treated in Boh (1993). Thus, in a recent study, Hanke (2022) examines the theories of Jerome Pardo (d. 1502), Gaspar Lax (1487–1560), and some further logicians from the "Brito-Italian tradition".

**4. St Augustine's epistemic logical principles and his conception of lying**
As mentioned already in section 1, Augustine introduces his investigations by remarking that not everybody saying something *false* must therefore be a liar. In particular, one does not lie if one says something which one *believes* to be true. The necessary and sufficient conditions for lying are elaborated as follows:

> Quisquis autem hoc enuntiat quod vel creditum animo, vel opinatum tenet, etiamsi falsum sit, non mentitur. Hoc enim debet enuntiationis suae fidei, ut illud per eam proferat, quod animo tenet, et sic habet ut profert. Nec ideo tamen sine vitio est, quamvis non mentiatur, si aut non credenda credit, aut quod ignorat nosse se putat, etiamsi verum sit: incognitum enim habet pro cognito. Quapropter ille mentitur, qui aliud habet in animo, et aliud verbis vel quibuslibet significationibus enuntiat. Unde etiam duplex cor dicitur esse mentientis, id est, duplex cogitatio: una rei eius quam veram esse vel scit vel putat, et non profert; altera eius rei quam pro ista profert sciens falsam esse vel putans. Ex quo fit ut possit falsum dicere non mentiens, si putat ita esse ut dicit, quamvis non ita sit; et ut possit verum dicere mentiens, si putat falsum esse et pro vero enuntiat, quamvis revera ita sit ut enuntiat. Ex animi enim sui sententia, non ex rerum ipsarum veritate vel falsitate mentiens aut non mentiens iudicandus est. Potest itaque ille qui falsum pro vero enuntiat, quod tamen verum esse opinatur, errans dici et temerarius: mentiens autem non recte dicitur; quia cor duplex cum enuntiat non habet, nec fallere cupit, sed fallitur. Culpa vero mentientis est, in enuntiando animo suo fallendi cupiditas; sive fallat cum ei creditur falsum enuntianti; sive non fallat, vel cum ei non creditur, vel cum verum enuntiat voluntate fallendi, quod non putat verum. Quod cum ei creditur, non utique fallit, quamvis fallere voluerit: nisi hactenus fallit, quatenus putatur ita etiam nosse vel putare ut enuntiat. (Augustinus (1900), p. 1, section 3)

> But who says something which he holds in his mind as believed or opined, does not lie even if it is false. For it is required by a pronunciation of one's belief, that he proffers by the pronouncement what he has in his mind, and that things are such as pronounced.[36] Yet, if someone believes what he should not believe, or if he thinks to know

[36] This imperative strongly reminds of Paul Grice's maxim of communication: 'Do not say what you believe to be wrong'. Cf. Grice (1975), p. 46.

> what he does not know, even if it be true, he is not without guilt, although he does not lie, for he holds something unknown for known. Therefore, someone is lying who has something different in mind than what he says with words or certain meanings. Thus, the liar is also said to have a forked tongue or a forked mind: either because he does not pronounce what he believes or knows to be true, or because he pronounces something which he knows or believes to be false. From this it follows that someone can say something false without lying, if he believes that things are such as he says they are, although they aren't such, and someone who says the truth can be a liar, if he believes to be false what he maintains to be true, while in fact things are such as he says. Thus, whether someone is lying or not has to be judged from the opinion of his mind, and not from the truth or falsity of the things themselves. Therefore, someone who truly pronounces something false, which he believes to be true, can be said to be *erring* and *unmindful*, but he must not really be called a liar, because he doesn't speak with a forked tongue, and he doesn't *want* to deceive, but he *is* deceived. But the guilt of a liar consists in the desire in his mind to deceive by speaking, no matter whether he deceives because something false is believed, or whether he does not deceive, either because he is not believed, or because, in the wish to deceive, he says something true, which he doesn't believe to be true. In case he is believed, he does not deceive, although he wanted to deceive; at best he only deceives in so far as it is believed that he knows or believes what he says.

In order to summarize and formalize the core of Augustine's theory of lying, or at least of the *epistemic* aspects of this theory,[37] let us introduce two further relations:

$S(a,p) =_{\mathrm{df}}$ *a says* that *p*
$L(a,p) =_{\mathrm{df}}$ *a* is *lying* by saying that *p*.

The elementary insight that *saying something false* does not automatically mean to *lie*, can be captured by the *invalidity* of principle

L1* $\quad S(a,p) \wedge \neg p \Rightarrow L(a,p)$.

[37] As a referee of this paper kindly pointed out, Augustine's theory has several other aspects, which cannot, however, be treated here. In particular Augustine emphasizes the liar's intention to *deceive* someone (and possibly to seduce him to do something wrong). A detailed discussion of such issues is to be found in Limonta (2024).

Instead, if someone says something *false* but *believes* it to be *true*, he is *not* lying:

L2 $S(a,p) \wedge \neg p \wedge B(a,p) \Rightarrow \neg L(a,p)$.

This is a corollary of the more general principle that if someone says something which he believes to be true, he is not lying:

L3 $S(a,p) \wedge B(a,p) \Rightarrow \neg L(a,p)$.

A *lie* obtains if and only if someone (intentionally) says the opposite of what he believes:

L4 $L(a,p) \Leftrightarrow (S(a,p) \wedge B(a,\neg p)) \vee (S(a,\neg p) \wedge B(a,p))$.

This principle is independent of the factual truth (or falsity) of *p*, i.e., in particular, if someone is saying something *true* while he believes it to be false, he is lying:

L5 $S(a,p) \wedge B(a,\neg p) \wedge p \Rightarrow L(a,p)$.

In the cases described by the antecedents of L2 and L5, subject *a* is *in error*, but only in case L5 *a* is also guilty of *lying*.

To conclude this paper, let us have a second look at Augustine's central epistemological statements:

> [1] Inter credere autem atque opinari hoc distat, [2] quod aliquando ille qui credit, sentit se ignorare quod credit, [3] quamvis de re quam se ignorare novit omnino non dubitet, si eam firmissime credit; [4] qui autem opinatur, putat se scire quod nescit.

In this short passage, Augustine uses two different notions ("scire", "nosse") for *knowing*; two different notions ("ignorare", "nescire") for *not-knowing*, three different notions ("credere", "opinari", "putare") for *believing*, and another expressing ("dubitare") for not-knowing or not-believing. According to principles E10, E11, *doubting that p* entails both $\neg K(a,p)$ and $\neg C(a,p)$, but the converse implication $\neg K(\mathrm{a},p) \rightarrow D(a,p)$ doesn't hold.

In sentence [1], Augustine announces a difference between 'credere' and 'opinari'. In [4] he says that 'opinari' entails a *belief to know*, ("putat se scire");

hence it appears plausible to interpret ‘opinari’ as our “strong” belief $C(a,p)$ which, according to “Moore’s principle” E12, entails $B(a,K(a,p))$. In contrast, ‘credere’ does not (always) entail a belief to know; this is expressed in sentence [2] where it is maintained that sometimes a belief obtains even if the person thinks (or “feels”) that she does not know what she believes (“sentit se ignorare quod credit”). Such a situation might be formalized by $B(a,p) \wedge B(a,\neg K(a,p))$, which is basically equivalent to $B(a,p) \wedge B(a,\neg C(a,p))$, and hence equivalent to $B(a,p) \wedge \neg C(a,p)$, i.e., a belief which is only “weak” but not “strong”. However, in [3] Augustine points out that in the case of the “strongest” belief (“firmissime credit”) about some “thing” $p$, which is at the same time characterized as the absence of any doubt concerning $p$ (“omnino non dubitet”), the subject *a knows* (!) that she doesn’t know (“se ignorare novit”) that $p$. This apparently flatly contradicts “Moore’s principle”, or more exactly, its corollary

$$\text{E13} \quad C(a,p) \rightarrow \neg K(a,\neg(K(a,p)),$$

which follows from E12 via the chain of epistemic/doxastic implications shown in E1. So, what Augustine *should* have said instead of [3] is rather: “quamvis de re quam se ignorare *nescit* omnino non dubitet, si eam firmissime credit”.

There is yet another incongruity in the text quoted from Augustine (1900). When, in [4], a “strong” belief (under the Latin expression ‘opinari’) is characterized as a *belief to know*, the concluding ‘quod nescit’ doesn’t make sense. For if ‘what is not known’ were added to ‘putat scire’, the latter belief would be disqualified as being *mistaken*. But this would mean that each instance of ‘opinari’ is a *false* belief. The logic of Augustine’s characterization of ‘opinari’ as a “strong” belief which constitutes a belief to know rather should have been formulated by saying: “qui autem opinatur, putat se scire quod *credit*”.

It seems desirable to inquire whether the manuscripts used by the editor of Augustine’s *De mendacio liber unus* really contain the two crucial expressions ‘novit’ and ‘nescit’ in the places where one would normally expect to read ‘nescit’ and ‘credit’. If it should turn out that the editior (or a scribe of the manuscript) made these mistakes, one could firmly maintain that St Augustine not only *anticipated*, but clearly *stated* “Moore’s principle” insofar as a *belief* that *p* entails a belief to know that *p* just in case that it is a "strong", or more exactly, the strongest possible belief ("firmissime credit").[38]

**5. Appendix: The re-discovery of "Moore's Principle" in the 20th century**

In order to outline the history of the re-discovery of "Moore's Principle" in the 20th century, it is necessary to distinguish various variants of this principle, namely in particular our former E4, which shall be renamed:

M1 $C(a,p) \rightarrow B(a,K(a,p))$,

and its strengthening into a bi-conditional, i.e., our former E5:

M2 $C(a,p) \leftrightarrow B(a,K(a,p))$.

Additional variants are obtained by either strengthening the operator of "weak" belief into "strong" belief:

M3 $C(a,p) \rightarrow C(a,K(a,p))$

M4 $C(a,p) \leftrightarrow C(a,K(a,p))$.

Alternatively, the "weak" belief operator in M2 can be further weakened, according to the chain of epistemic/doxastic entailments E1, as follows:

M5 $C(a,p) \leftrightarrow \neg B(a,\neg K(a,p))$

M6 $C(a,p) \leftrightarrow \neg C(a,\neg K(a,p))$

M7 $C(a,p) \leftrightarrow \neg K(a,\neg K(a,p))$.

It is not easy to tell who coined the label 'Moore's principle'. According to Harrison (1969), law M1 was suggested by G. E. Moore in his (1950) paper "Certainty", but Harrison did not call this law 'Moore's Principle'.[39] As reported in Lenzen (2003), p. 23, Vincent F. Hendricks pointed out to me that "Lamarre/Shoham [1994] and other recent authors refer to [M3] as 'Moore's

[38] I am grateful to Roberto Limonta for having pointed out to me that the edition Augustine (1900) is based on a manuscript preserved at the Staatsbibliothek Munich which is online available at https://www.digitale-sammlungen.de/de/view/bsb00046468?q=%28M+14431%29&page=130,131. I have thus been able to verify that the crucial "mistakes" are not due to the editor of Augustine (1900), and it only remains to be investigated whether perhaps a scribe made the "mistakes".

[39] Cf. Harrison (1969), p. 87, note 2: "The proposal that certainty be taken to mean belief that one knows was made by G. E. Moore. See his article 'Certainty', in *Philosophical Papers*".

principle because the basic idea that being certain entails being certain that one knows is thought to have first been put forward in [Moore 1950]”.[40] As a matter of fact, however, Lamarre & Shoham did not use the *expression* ‘Moore’s principle’ either, even though they clearly endorsed several variants of this law for their own epistemic logic.

As regards the question whether M1 or M3 may rightly be attributed to Moore at all, Lenzen (2003), p. 30, criticized that “[…] Moore all too often appears to conflate the (semantic) truth-conditions for $C(a,p)$ and $K(a,p)$ on the one hand and the pragmatic conditions for the utterability of the corresponding assertions ‘I am certain that *p*’ and ‘I know that *p*’ on the other hand.” Nevertheless, it seems justified to attribute to Moore at least a vague knowledge of principle M3, for he maintained in (1950), p. 266: “[…] if anybody asserts ‘It is certain that *p*’ part of what he is asserting is that he himself knows that *p* is true”. However, there is no evidence that Moore ever endorsed the *equivalence* M4, or the more sophisticated principles M1, M2.

According to Harrison (1969), “James and Dewey also construed certainty as belief that one knows, probably deriving the idea from Peirce”, but the author failed to provide further information where “James’ Principle” or “Dewey’s Principle” might possibly have been stated. Furthermore, he describes “Peirce’s Principle” as the idea that “perfect certainty be interpreted as *knowledge* [!] that one knows”. This principle, however, basically *equates* certainty with knowledge and is hence much stronger than any variant of “Moore’s Principle”.

As was argued in Lenzen (1978), the vast majority of “Recent Work in Epistemic Logic” (where “recent” meant ‘published in the 1960ies and 1970ies’) suffered from not sufficiently distinguishing between “weak” and “strong” belief. This verdict also applies to Hintikka who only stated that – as a simple corollary of the “KK-thesis” K4 – *knowing* that *p* entails believing that one knows that *p* (cf. Hintikka (1962), p. 50). But he nowhere discussed the question whether believing to know is also entailed by a (“strong” or “weak”) *belief*.

The first clear pronouncements of “Moore’s principle” (though not under this label) apparently may be found in Ulrich Blau’s doctoral dissertation *Glauben und Wissen* of 1968/69, and in Franz von Kutschera’s *Einführung in die intensionale Semantik* (1976). In Lenzen (1978), p. 164, it was maintained

[40] Similarly, in Lenzen (2012), p. 311, M1 was called ‘Moore’s principle’ because, according to Hendricks (2004), Lamarre & Shoham used this label.

that these two works contain law M7 as a theorem. As a matter of fact, however, Kutschera's system only lists the weaker theorem M3.[41]

The validity of principles M1 and M3 was first stated in Lenzen (1978), p. 80, where it was also pointed out that these implications become invalid if the premise $C(a,p)$ is weakened to $B(a,p)$. The *equivalences* M2 and M4 plus their variants M5 – M7 have been discussed in section 3.3 of Lenzen (1980).[42] In both works it was further pointed out that principle M7, i.e. our earlier E6, is systematically very important for the exact determination of the axiomatic structure of epistemic logic. As proven in Lenzen (1979), the epistemic analogue of the characteristic logic of calculus **S 4.2**, $\Diamond\Box p \to \Box\Diamond\Box p$, i.e., $\neg W(a,\neg W(a,p)) \to W(a,\neg W(a,\neg W(a,p)))$, can be transformed by means of M7 into the simpler principle $C(a,p) \to K(a,C(a,p))$, i.e., our earlier D4. Hence epistemic logic is at least as strong as **S 4.2**. Furthermore, the epistemic analogue of the characteristic logic of **S 4.4**, $p \to (\Diamond\Box p \to \Box p)$, i.e., $p \to (\neg W(a,\neg W(a,p)) \to W(a,p))$, amounts to the claim that if $p$ is true and if $a$ is convinced that $p$, $a$ knows that $p$. Hence this principle characterizes the logic of 'knowledge' as 'true conviction'. In Lenzen (2012), it was further emphasized that, in view of M7, there are exactly twelve different, irreducible epistemic/doxastic modalities which can be arranged into three nested squares of opposition.[43]

## 6. References

AUGUSTINUS: *De mendacio liber unus*, in *Sancti Augustini Opera*, Sectio V, Pars III (CSEL 41), ed. by J. Zycha, Prague: Vindobonae Lipsiae, 1900, 411–466.

[41] Cf. Kutschera (1976), p. 94, theorem **TG3**e. Kutschera's epistemic operators are mostly symbolized without mention of the respective subject, and 'GA' (abbreviating 'A wird geglaubt') is to be understood as a "strong" belief. Principle M3 is proved by means of the definition of knowledge as true belief.

[42] In this work the attitudes of believing, being convinced, and knowing are symbolized by '$G(a,p)$' ("*a* glaubt, dass *p*'), '$Ü(a,p)$' ("*a* ist davon überzeugt, dass *p*") and '$W(a,p)$' ("*a* weiß, dass *p*"), respectively.

[43] Some variants of "Moore's Principle" have been "re-invented" in Lamarre & Shoham (1994), p. 415: "1. One should distinguish 'knowing', 'being certain', and 'believing'. Knowledge entails certainty which entails belief, but there is more structure to the three notions than mere hierarchy [as summarized in our E1]. The intuitions behind 'certainty' is that, to the agent, the facts of which he is certain appear to be knowledge; there is no such connection between [belief and knowledge]. Thus, 'John is certain that' is equivalent to 'John is certain that John knows that' [as expressed in our M4], but 'John believes that' is not equivalent to 'John believes that John is certain that', and definitively not to 'John believes that John knows that'. (In fact, in our system 'John believes that John knows that' will turn out to be equivalent to 'John is certain that' [i.e., our M2])."

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