Joint Obligations

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Abstract

Obligations are addressed to persons and require that they do something, refrain from doing something, prevent something or see to it, that a certain state of affairs is realized or preserved. Therefore a theory of action is the appropriate frame for deontic logic. The frame for such a theory is the logic of branching histories ($T \times W$ logic), a combination of tense and modality, to which alternatives for persons are added. In a paper on collective alternatives (2014) I have shown that the alternatives for groups of agents do not always derive from the alternatives of their members. In this paper I want to examine the consequences for deontic logic. Its largest part, however, is about the action-theoretic preliminaries. Readers familiar with them may turn directly to the last paragraph.

1 Introduction

To indicate the place of this paper in the field of deontic logics I begin with a brief historical review. The pioneer of deontic logic was Georg Henrik von Wright. In his paper (1951) he has formulated the first modern system of monadic deontic logic. The usual possible-worlds semantics was added later by J. Hintikka, S. Kanger, R. Montague and S. Kripke.

In a paper from 1963 Roderick Chisholm maintained that monadic deontic logics fail in the face of contrary-to-duty imperatives: That B ought to be if A, can only be expressed in them either by (a) $A \supset O(B)$ or by (b) $O(A \supset B)$. Let A be: Max violates some laws and B: Max is punished. Then B ought to be if A, and $\neg B$ ought to be if $\neg A$. Let it be the case that A. If we take the alternative (a) we obtain for $O(\neg A)$ the allegedly absurd obligation $O(\neg A \wedge B)$: It ought to be the case that Max does not violate any laws and Max is punished. If we take the alternative (b) we obtain from $O(\neg A)$ for all C: $O(A \supset C)$, i.e. contrary-to-duty behavior implies arbitrary conditional obligations. It was again von Wright who in (1964) and (1965) first proposed a dyadic deontic logic for which conditional obligations, expressed by sentences of the form $O(B, A)$, are basic. I do not consider Chisholm's example a more serious objection than Ross's paradox $O(A) \supset O(A \vee B)$, however. It certainly sounds strange to say that since it is obligatory to love one's neighbor it is also obligatory to love or kill him, but you cannot love him without loving or killing him, after all, and you cannot fulfil a set of norms by fulfilling just some of them. In Chisholm's paradox Max in fact violates some law in the world w in which the obligations are to hold, and should therefore be punished, $O(B)$. In w it is also obligatory that Max violate no law, $O(\neg A)$. Therefore we have $O(\neg A \wedge B)$: It is obligatory that Max violate no law and also be punished, since he has in fact not fulfilled his obligations. This conclusion sounds strange only if you read it as "It should be the case that Max does not violated any laws but still is punished." Since the truth conditions for obligations hold relatively to worlds, i.e. with respect to different conditions, the idea of dyadic deontic logic to state the conditions also explicitly is not very convincing. Therefore I shall ignore conditional obligations in this paper.

It was von Wright, once more, who made the distinction between norms to be and norms to do. A norm to do is addressed to persons and demands that they do something or refrain from doing something. A norm to be like "No parking in this yard" is not addressed to specific persons but still to people. Persons are obliged or forbidden to do something. Therefore obligations should generally be formulated in the frame of an action theory. The arguments of von Wright in (1974) for a combination of action theory with deontic logic were mainly that sentences of the form "X ought to do F" or "X shall do F" often have a prescriptive sense and therefore no truth value. He tried to develop a logic of prescriptions, but that is quite another thing than a deontic logic which states obligations and should be treated separately.

2 $T \times W$ logic as a frame for action theory

2.1 Actions

An action of a person is a behaviour she could also have refrained from. Every action arises from a situation in which the agent has at least two alternatives. If we describe a behaviour of a person as an "action" we presuppose that she could have done otherwise. That is not always the case if it was possible that she would behave differently. If somebody falls down the stairs for instance, that is normally a contingent event. Therefore it was possible that he would not fall. But this does not make his fall an action. In the case of an action it has to be possible for the agent to do otherwise and this possibility must consist in an alternative that was open to him.[1] We therefore distinguish between objective or event possibility and subjective or agent possibility, between what is possible in view of the laws of nature e. g., and what is possible for an agent. This important distinction was first made by Thomas Aquinas.[2]

2.2 $T \times W$ frames

The appropriate frame for a theory of actions is a combination of modal and tense logic as $T \times W$ logic, the logic of branching worlds (or histories). Here I presuppose the same time ordering for all worlds and, for the sake of simplicity, also a discrete time structure so that for every moment, but the last, there is a next moment.

$T \times W$ frames are defined as in R. Thomason (1984):

[1] There is a whole library on the relation between the statements "It is possible for the agent X to do F" and "It is possible, that X does F", and between "X could have done otherwise" and "If X would have wished differently he would have acted differently". Determinists naturally misinterpret the first sentences in the sense of the latter. Cf. e. g. J. J. Smart (1963), chapter 6.

[2] De proprietates mod., zit. in Bochenski (1956), p. 211 seq.

Definition 1 *(D1) A* $T \times W$ *frame is a quadruple* $U = (T, <, W, \sim)$, *where*

1) T *is a nonempty set of time points,*

2) $<$ *is a linear ordering on* T,

3) W *is a nonempty set of worlds (or histories), and*

4) for all $t \in T$ $\sim_t$ *is an equivalence relation on* W *such that* $w \sim_t w'$ *and* $t' < t$ *implies* $w \sim_{t'} w'$.

$W(t, w)$ is to be $\{w' : w' \sim_t w\}$, the set of worlds that are possible as seen from t and w. Propositions A, B, ... are subsets of $T \times W$. A is *analytically necessary* – in symbols $\Box A$ – in t and w iff A holds in all worlds w' at t. A is *historically necessary* – in symbols NA – in t and w iff $W(t, w) \subseteq A_t$, where $A_t = \{w : (t, w) \in A\}$.

3 The logic of actions

3.1 Separate alternatives

The starting points for a logic of actions have been G. H. von Wright (1967) and L. Åqvist in (1974). In Kutschera (1986) and (1993) alternatives for several agents were discussed, and joint alternatives in Kutschera (2014).

To represent momentary actions in a $T \times W$ frame U we add a finite set $G = \{g_1, ..., g_n\}$ of agents:

Definition 2 *(D2) A system of separate alternatives based on* $U = (T, <, W, \sim)$ *is a pair* (G, A) *such that:*

a) G *is a set of agents,* $g_1, ..., g_n$.

b) For all g, w, t $A(g, t, w)$ *is the set of alternatives of the agent* g *in* w *and* t. *These sets have the following properties:*

b1) $w' \in W(t, w) \supset A(g, t, w) = A(g, t, w')$.

b2) $X \in A(g, t, w) \supset \varnothing \neq X \subseteq W(t, w)$.

b3) $w' \in X \wedge X \in A(g, t, w) \supset W(t + 1, w') \subseteq X$.

b4) $X, Y \in A(g, t, w) \supset X = Y \vee X \cap Y = \varnothing$.

b5) $W(t, w) \subseteq \cup A(g, t, w)$.

b6) $X_1 \in A(g_1, t, w) \wedge ... \wedge X_n \in A(g_n, t, w) \supset X_1 \cap ... \cap X_n \neq \varnothing$.

b7) $w' \in W(t, w) \supset \exists X_1 ... X_n (X_1 \in A(g_1, t, w) \wedge ... \wedge X_n \in A(g_n, t, w) \wedge X_1 \cap ... \cap X_n = W(t + 1, w'))$.

Remarks: (b1) Alternatives do not depend on the future. (b2, b4, b5) The sets of individual alternatives in w at t are partitions of the set $W(t, w)$ of possible worlds. (b3) Agents cannot discriminate in t between worlds that branch only later than t. (b6) No alternative can be blocked by choices of the other agents. (b7) All the agents together can determine how the world goes on. The set G of agents, then, must contain everyone and everything that has an influence on future developments after any branching point. Therefore often one of them will be Mother Nature, who is responsible for chance events.

Not every agent has a choice at every moment. Therefore sets of alternatives $A(g, t, w)$ are admitted containing $W(t, w)$ as the only alternative. In this case I shall say that g has no genuine alternative, no alternative he could refrain from realizing. An agent has a genuine alternative only if he has at least two alternatives.

With respect to these alternatives we can define agent possibility: For the agent g it is possible in w at t to bring about the state of affairs A, if g has a genuine alternative in w, t for which A holds at t in all the worlds of this alternative. An agent g brings it about in w at t that A holds at t, if the alternative g realizes in w at t is a subset of the set of all worlds in which A holds at t.

In this framework we can now define objective and subjective possibilities: In w, t it is objectively (historically) possible that a state of affair A obtains in t, if $W(t, w) \cap A_t \neq \emptyset$. In w, t it is possible for the agent g to bring it about that A, if there is an alternative $Y \in A(g, w, t)$ such that $Y \subseteq A_t$.

3.2 Joint alternatives

Alternatives of groups of agents from G are usually defined by individual alternatives: If $A(\{g_{i_1}, ..., g_{i_m}\}, t, w)$ is the set of alternatives of the group $\{g_{i_1}, ..., g_{i_m}\}$ in w, t we have:

Definition 3 *(D3)* $A(\{g_{i_1}, ..., g_{i_m}\}, t, w) :=$
$\{X_1 \cap ... \cap X_m : X_1 \in A(g_{i_1}, t, w) \wedge ... \wedge X_m \in A(g_{i_m}, t, w)\}$.

The alternatives of a group, therefore, are the combinations of the individual alternatives of its members.

According to D3 the alternatives of a group result from the alternatives of its members. In realizing a joint alternative they do in coordination, what they can also do separately. There are, however, many cases in which groups have new possibilities, possibilities beyond those envisaged by D3. The following examples show that co-operation opens up new possibilities.

Case 1: Two mountaineers can either climb peak B separately, a lower pinnacle in front of peak A, or they can climb A together, as a team. Each of them has two individual alternatives – to stay in the camp or to climb peak B – but together they have the additional alternative of climbing A as a team. This alternative does not arise from the separate possibilities in the way stated in D3.

Case 2: In Ruritania prison cells for two occupants are so small that there is only room for one person to sit while the other has to stand. The occupants of such a cell have no genuine individual alternative. They cannot sit or stand independently of what the other does, so that, without co-operation, their positions will have to remain as they are. Only in a coalition they have genuine alternatives and can determine, who sits and who stands. These collective alternatives again do not result from individual ones.

The following example shows that the alternatives of individuals or groups may also depend on coalitions between other agents:

Case 3: John, Tom and Max each want to have what is left in a bottle of rum. John is stronger than each of the other two but together they can hold him back. So if Tom and Max cooperate John has no alternative, but if there is no co-operation between Tom and Max, John may drink the rest of the rum or leave it to the others, as he pleases. His alternatives depend on the co-operation between the others.

Such examples suggest that we conceive of joint alternatives not as combinations of individual alternatives as in D3 but as fundamental and define them relatively to coalitions among the rest of the agents. Coalitions are specified by partitions $D = \{G_1, ..., G_m\}$ of the set G of agents. So we consider sets of alternatives $A(G_i, D, t, w)$ for $G_i \in D$. For individual alternatives we have $A(g, t, w) = A(\{g\}, D_0, t, w)$ for $D_0 = \{\{g_1\}, ..., \{g_n\}\}$, and for the collective alternatives of D3 $A(\{g_{i_1}, ..., g_{i_m}\}, t, w) = A(\{g_{i_1}, ..., g_{i_m}\}, (D_0 - \{g_{i_1}\}, ..., \{g_{i_m}\}) \cup \{g_{i_1}, ..., g_{i_m}\}, t, w)$.

If we consider the groups $G_1, ..., G_m$ in a partition D of G as individuals we get conditions corresponding to those of D2. The main difference is that the alternatives in $A(G_i, D, t, w)$ are partitions not of $W(t, w)$, the set of all worlds possible in w, t, but of a non-empty subset $W(D, t, w)$ of $W(t, w)$, the set of possible outcomes for coalition structure D. Without co-operations in our example 1 the climbing of peak A, in example 2 the prisoners changing positions, and in example 3 Tom and Max getting some of the rum is not a possible outcome.

Definition 4 *(D4) A system of joint alternatives based on the $T \times W$ frame U is a pair (G, A) such that:*

a) G is a set of agents, $g_1, ..., g_n$.

b) For all partitions $D = \{G_1, ..., G_m\}$ of G and all w, t $A(G_i, D, t, w)$ is the set of alternatives of the group G_i relative to the partition D in w and t. For $W(D, t, w) := \cup_{1 \leq i \leq m} A(G_i, D, t, w)$ these sets have the following properties:

b1) $W(D, t, w) \subseteq W(t, w)$.

b2) $w' \in W(t, w) \to A(G_i, D, t, w) = A(G_i, D, t, w')$.

b3) $X \in A(G_i, D, t, w) \to \varnothing \neq X \subseteq W(D, t, w)$.

b4) $w' \in X \wedge X \in A(G_i, D, t, w) \to W(t+1, w') \subseteq X$.

b5) $X, Y \in A(G_i, D, t, w) \to X = Y \vee X \cap Y = \varnothing$.

b6) $X_1 \in A(G_1, D, t, w) \wedge ... \wedge X_m \in A(G_m, D, t, w) \to X_1 \cap ... \cap X_m \neq \varnothing$.

b7) $X \in A(G_i, D, t, w) \wedge Y \in A(G_k, D, t, w) \to \exists Z(Z \in A(G_i \cup G_k, (D - \{G_i, G_k\}) \cup \{G_i \cup G_k\}, w, t) \wedge Z \subseteq X \cap Y)$.

b8) $w' \in W(D, t, w) \to W(t+1, w') \in A(G, \{G\}, t, w)$.

Remarks: (b2−7) are taken over from D2. (b7) corresponds to D3; example 2 shows that $Z \subseteq X \cap Y$ cannot be replaced by $Z = X \cap Y$. From (b7) we get

c) $X \in A(G_i, D, t, w) \to \exists Y(Y \in A(G, \{G\}, t, w) \wedge Y \subseteq X)$ − the biggest coalition $\{G\}$ can bring about everything that smaller coalitions can bring about. (b8) is the completeness condition corresponding to D3, b7.

If we count Mother Nature, n, among the agents we should only consider coalition structure D such that $\{n\} \in D$ since there can be no cooperation with chance.

From (b8) we obtain $W(t, w) \subseteq W(G, \{G\}, t, w)$, and in view of (b1)

d) $W(t, w) = W(G, \{G\}, t, w)$ and

e) $W(D, t, w) \subseteq W(G, \{G\}, t, w)$.

3.3 Strategies and actions

Actions of individuals or coalitions mostly do not consist in the realization of a momentary alternative but in following a strategy that determines what to do now and what to do next in each case that may arise from the present choice. I shall not define strategies as sequences of momentary alternatives, however, but directly. First separate strategies of individuals:

Definition 5 *(D5) A system of separate strategies based on the $T \times W$ frame U is a pair (G, S), such that:*

a) G is a set of agents, $g_1, ..., g_n$.

b) For $g \in G$, $w \in W$ and $t \in T$ $S(g, t, w)$ is the set of the strategies of g in w, t. These sets have the following properties:

b1) $w' \in W(t, w) \supset S(g, t, w') = S(g, t, w)$.

b2) $X \in S(g, t, w) \supset \varnothing \neq X \subseteq W(t, w)$.

b3) $X, Y \in S(g, t, w) \supset X = Y \vee X \cap Y = \varnothing$.

b4) $W(t, w) \subseteq \cup S(g, t, w)$.

b5) $X_1 \in S(g_1, t, w) \wedge ... \wedge X_n \in S(g_n, t, w) \supset X_1 \cap ... \cap X_n \neq \varnothing$.

b6) $w' \in W(t, w) \wedge t < t' \supset \exists X_1 ... X_n (X_1 \in S(g_1, t, w) \wedge ... \wedge X_n \in S(g_n, t, w) \wedge X_1 \cap ... \cap X_n \cap W(w', t') \neq \varnothing)$.

b7) $w' \in W(t, w) \wedge t < t' \supset S(g, t', w') = \{X \cap W(t', w') \neq \varnothing : X \in S(g, t, w)\}$.

Actions are realized by strategies, but often the same action may be realized in different ways, by different strategies. Then it has to be represented by a union of strategies. Hence $H(g, t, w) = \{X_1 \cup ... \cup X_m \neq W(t, w) : X_1, ..., X_m \in S(g, t, w)\}$ is the set of possible actions of the agent g in w. An action $X \in H(g, t, w)$ is finite iff for all $w' \in X$ there is a time t': $t < t'$, such that $W(t', w) \subseteq X$.

To express that an agent g does something or causes something we introduce two operators D and C.

Definition 6 *(D6) a) $D_g(A) := \{(t, w) : A_t \in H(g, t, w) \wedge w \in A_t\}$ – g does A.*

b) $C_g(A) := \{(t, w) : \exists X (X \in H(g, t, w) \wedge X \subseteq A_t \wedge w \in X)\}$ – g brings it about that A.

c) $OD_g(A) := M(D_g(A)) \wedge \neg A$ – g omits to do A.[3]

d) $OC_g(A) := M(C_g(A)) \wedge \neg A$ – g omits to bring it about that A.

e) $P_g(A) := C_g(\neg A)$ – g prevents that A.

f) $M_g := C_g(T \times W)$ – g has a choice.

"It is possible for g to do A" may be expressed by $M(D_g(A))$. It is true in t, w iff $A_t \in H(g, t, w)$. Likewise for "It is possible for g to bring it about that A". Since $D_g(A)$ has an active sense, a meaning which implies that g can refrain from doing A, we can express subjective as well as objective possibility with the operator M.

For separate alternatives $H(\{g_{i_1}, ..., g_{i_m}\}, t, w) = \{X_1 \cap ... \cap X_m : X_1 \in H(g_{i_1}, t, w) \wedge ... \wedge X_m \in H(g_{i_m}, t, w)\}$ is the set of possible actions of the group $\{g_{i_1}, ..., g_{i_m}\} \subseteq G$.

Joint strategies and actions may be defined in the same way:

[3] Since $D_g(A)$ etc. are not sentences but propositions I should write $\cap$ instead of $\wedge$ and $T \times W - A$ instead of $\neg A$ but the sentential connectives make easier reading.

Definition 7 *(D7) A system of joint strategies based on the* $T \times W$ *frame* U *is a pair* (G, S) *such that:*

a) G *is a set of agents,* $g_1, ..., g_n$.

b) For all partitions $D = \{G_1, ..., G_m\}$ *of* G *and all* w, t $S(G_i, D, t, w)$ *is the set of strategies of the group* G_i *of agents relatively to a coalition structure* D *in* w *and* t*. These sets have the following properties:*

b1) $W(D, t, w) \subseteq W(t, w)$.

b2) $w' \in W(t, w) \supset S(G_i, D, t, w) = S(G_i, D, t, w')$.

b3) $X \in S(G_i, D, t, w) \supset \varnothing \neq X \subseteq W(D, t, w)$.

b4) $X, Y \in S(G', t, w) \supset X = Y \vee X \cap Y = \varnothing$.

b5) $X_1 \in S(G_1, D, t, w) \wedge ... \wedge X_m \in S(G_m, D, t, w) \supset X_1 \cap ... \cap X_m \neq \varnothing$.

b6) $X \in S(G_i, D, t, w) \wedge Y \in S(G_k, D, t, w) \supset \exists Z(Z \in S(G_i \cup G_k, (D - \{G_i, G_k\}) \cup \{G_i \cup G_k\}), t, w) \wedge Z \subseteq X \cap Y)$.

b7) $w' \in W(t, w) \wedge t < t' \supset \exists X(X \in S(G, \{G\}, t', w') \wedge X \cap W(t', w') \neq \varnothing)$.

b8) $w' \in W(D, t, w) \wedge t < t' \supset$
$S(G_i, D, t', w') = \{X \cap W(D, t, w) \neq \varnothing : X \in S(G_i, D, t, w)\}$.

Actions of groups may be defined as above and also the operators $D_{G'}$ and $C_{G'}$. For different agents we have also principles like $M(D_g(A)) \supset \neg M(P_{g'}(A)), M(D_{g_1}(A_1)) \wedge ... \wedge M(D_{g_m}(A_m)) \supset M(D_{\{g_1,...,g_m\}}(A_1 \wedge ... \wedge A_m))$.

4 Obligations

4.1 Separate obligations

Let $P(t, w)$ be the set of permissible worlds in $W(t, w)$. Then we have $\varnothing \neq P(t, w) \subseteq W(t, w)$ and we may define obligations by

Definition 8 *(D8)* $O(A) := \{(t, w) : P(t, w) \subseteq [A]_t\}$ *– It is obligatory that A.*

With O we can express obligations to be – obligations that something should be the case – as well as obligations to do. $O(D_g(A))$ is the proposition that the agent g should do A, that it is obligatory for g to do A. Because of $O(A) \supset M(A)$ (according to D8) we have $O(D_g(A)) \supset M(D_g(A))$ – Ought implies Can.

4.2 Joint obligations

Let $R = (U, G, S)$ be a system of separate strategies in the sense of D5. Then obligations of groups of agents correspond to obligations of their members: They ought to do as individuals what they ought to do in the group. This is not true anymore if we consider systems of joint strategies in the sense of D7, and that is the main point of my paper. $W(D, t, w) \subseteq W(t, w)$ is again the set of possible outcomes of the alternatives of groups for the partition D of the set G of all agents. We may have different sets $P(D, t, w)$ of permissible worlds for different partitions D. We must have $\varnothing \neq P(D, t, w) \subseteq W(D, t, w)$. For $P(t, w) \cap W(D, t, w) \neq \varnothing$ we may set

$P(D,t,w) = P(t,w) \cap W(D,t,w)$. Otherwise we should not say that in case of D anything goes, but that D is forbidden. If in example 1 of 3.2 the joint climbing of peak A by the two mountaineers is the only permissible procedure − let us say that they have been paid to climb peak A − and therefore obligatory, they ought to co-operate.

Hence there may be obligations to form coalitions. A general approach would have to evaluate coalition structures by the utility of their outcomes and to give the highest value to a coalition that can attain most. That, in fact, would always be a coalition of all the agents. To permit only such coalitions would, however, be inadequate in many cases. Often we have to implement our framework by valuations for the outcomes of separate and joint actions. Sometimes it will also be better to use expected values, and then we need probabilities for sets of worlds. The more realistic the applications of our logical instruments are to be the more complex they become, and there will come a point where we have to ask if the results really justify the expense. Do we get a better grasp of complex situations by using more and more complex logical instruments for its analysis? That is a problem for all philosophical logics. Often we have to be content if we can define important conceptual differences and connections in simple models, as in our case in models of individual and joint agency, in which we may distinguish between objective and subjective possibilities, omitting and preventing, study the dependence of alternatives on coalitions and determine the corresponding obligations.

References

[1] Åqvist, L. (1974). A new approach to the logical theory of actions and causality. In S. Stenlund (ed.). (1974). *Logical Theory and Semantical Analysis*. Dordrecht: Reidel.

[2] Bochenski, J. M. (1956). *Formale Logik*. Freiburg i.B.: Alber.

[3] Chisholm, R. M. (1963). Contrary-to-duty imperatives and deontic logic. *Analysis* 24, pp. 33−36.

[4] Kutschera, F. v. (1986). Bewirken. *Erkenntnis* 24, pp. 253−281.

[5] Kutschera, F. v. (1993). Causation. *Journal of Philosophical Logic* 22, pp. 75−88.

[6] Kutschera, F. v. (2014). Collective alternatives. In R. Ciuni, H. Wansing, C. Willkommen (eds.) (2014). *Recent Trends in Philosophical Logic*. Heidelberg: Springer, pp. 139−144 (Trends in Logic 41).

[7] Smart, J. J. (1963). *Philosophy and Scientific Realism*. London: Routledge and Kegan Paul.

[8] Thomason, R. H. (1984). Combinations of Tense and Modality. In D. M. Gabbay and F. Guenthner, (eds.) (1984). *Handbook of Philosophical Logic* 2, pp. 135−165, (2nd edition 7, 2002, pp. 205−234).

[9] Wright, G. H. v. (1951). *An Essay in Modal Logic*. Amsterdam.

[10] Wright, G. H. v. (1964). A new system of deontic logic. *Danish Yearbook of Philosophy* 1, pp. 173−182.

[11] Wright, G. H. v. (1965). A correction to ‘A new system of deontic logic’. *Danish Yearbook of Philosophy* 2, pp. 103−107.

[12] Wright, G. H. v. (1967). The logic of action. In N. Rescher (ed.) (1967). *The Logic of Decision and Action*. Pittsburgh: University of Pittsburgh Press, pp. 121−136.

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